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Maximal Linked Systems and Ultrafilters in Abstract Attainability Problem ⋆

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Abstract: The given investigation is oriented to study of generalized elements (GE) for solving problems of attainability under constraints of asymptotic character. But, the development of this direction required a special study of the issues connected with structure of the GE themselves. In considered problems, GE are used for extension of the space of usual solutions or usual controls. This extension has an analogy with extension of topological spaces (TS). So, we use compactification procedures. It is important to know the possibilities for realization of the corresponding compactification of the initial solution space. This investigation is directed at this work. We consider topological constructions realized by ultrafilters of widely understood measurable spaces.

Keywords: filter, Stone space, topology, ultrafilter, Wallman extension.

1. INTRODUCTION

In control problems, not infrequently, stability with respect the constraints weakening be lacking. Namely, small weakening of constraints generates (in this case) spasmodic variation of attainable result. In addition, in extremal problem, under small weakening of constraints, spasmodic improvement of the criterion values is possible. In attainability problem, the spasmodic extension of attainability domain can be realized. We consider the last variant of a problem. And what is more, we investigate the essential generalization of the above-mentioned attainability problem. Namely, the abstract attainability problem with constraints of asymptotic character is considered. In addition, constraints is defined by nonempty family of sets. For investigation of such problems, constructions with employment of generalized elements (generalized controls) are required.

General questions connected with employment of generalized controls are considered in J.Warga (1972); R.V.Gamkrelidze (1977); N.N.Krasovskii (1968) and in many other publications. Very often, generalized elements generating corresponding solutions are defined as measures or measure-valued functions (see (J.Warga , 1972, ch.III,IV) and R.V.Gamkrelidze (1977)). In this case, we have generalized controls defined as measures. Recall that in A.G.Chentsov (1996, 1997); A.G.Chentsov and S.I.Morina (2002), finitely additive measures (see N. Danford and J.T.Shwarts (1958)) were used for construction of generalized elements and, in particular, generalized controls. This variant corresponds (in idea) to dynamic problems with impulse constraints. For such problems, we note original approach of N.N.Krasovskii connected with application of distributions.

On informative level, we consider one simplest example fixing the system

$$\dot{x}(t) = u(t), \ \ 0 \le t \le 1; \tag{1}$$

($x(t)$ and $u(t)$ are scalars), suppose that $x(0) = 0$. In addition, $u(\cdot) = (u(t), \ 0 \le t < 1)$ is piecewise constant and continuous from the right real-valued function. Here, we denote the set of all such functions by $\mathbf{U}_0^1$. We denote by $\mathbf{x}(u(\cdot))$ trajectory of the system (1):

$$\mathbf{x}(u(\cdot))(\theta) = \int_0^\theta u(t)dt \ \ \forall \theta \in [0,1].$$

Introduce the following phase constraint: $|\mathbf{x}(u(\cdot))(t)| \ge 1 \ \ \forall t \ \in]0,1]$. Consider the question about attainability domain in time 1 under validity of this constraints. Of course, we obtain empty set since admissible controls are lacking. So, our problem with precise constrains is empty. But, introduce the sets

$$\tilde{\mathbf{U}}_0^1[\theta] = \{u(\cdot) \in \mathbf{U}_0^1 | |\mathbf{x}(u(\cdot))(t) \ge 1 \ \forall t \in [\theta,1]\} \ \forall \theta \in]0,1].$$

Let $\tilde{\mathcal{U}} = \{\tilde{\mathbf{U}}_0^1[\theta] \ : \ \ \theta \in]0,1]\}$. Consider the family $\tilde{\mathcal{U}}$ as (distinctive) constraints of asymptotic character. We can associate the set $[-2,-1] \cup [1,2]$ to every number $\theta \ \in]0,1[$ as new variant of attainability domain (more exactly, attainability domain under weakened constraints). Of course, this set can be considered and as the limit set

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or as attraction set under our "asymptotic" constraints. In the following, we investigate such attraction sets under very general suppositions. For this, extension constructions are used. In these constructions, different variants of generalized elements are required.

Under investigation of generalized element spaces, it is becomes clear that procedures using such elements in control problems assume natural analogies with extension of TS (see R. Engelking (1977); R.A.Aleksandrjan and E.A.Mirzachanjan (1979); A.V.Arhangelskii (1989) and others). In this investigation, some analogies of such type are considered (constructions similar Wallman extension are discussed).

2. GENERAL NOTIONS AND DESIGNATIONS

We use the standard set-theoretical symbolics; $\emptyset$ is empty set and $\triangleq$ is the equality by definition. A family is the set all elements of which are sets too. By $\mathcal{P}(X)$ (by $\mathcal{P}'(X)$) we denote the family of all (all nonempty) subsets of a set X; moreover, by $\mathrm{Fin}(X)$ we denote the family of all finite sets of $\mathcal{P}'(X)$. For any set $\mathbf{M}$ and a family $\mathcal{M} \in \mathcal{P}'(\mathcal{P}(\mathbf{M}))$, suppose that

$$\mathbf{C_M}[\mathcal{M}] \triangleq \{\mathbf{M} \setminus M : \ M \in \mathcal{M}\} \in \mathcal{P}'(\mathcal{P}(\mathbf{M})).$$

Moreover, for any nonempty family $\mathcal{A}$ and a set B, we obtain that

$$\mathcal{A}|_B \triangleq \{ \ A \cap B : \ A \in \mathcal{A}\} \in \mathcal{P}'(\mathcal{P}(B)).$$

For any nonempty family $\mathbf{X}$, we suppose that

$$(\{\cup\}(\mathbf{X}) \triangleq \{ \bigcup_{X \in \mathcal{X}} X : \ \mathcal{X} \in \mathcal{P}(\mathbf{X})\}) \ \& \ (\{\cap\}_\sharp(\mathbf{X}) \triangleq \{ \bigcap_{X \in \mathcal{K}} X : \ \mathcal{K} \in \mathrm{Fin}(\mathbf{X})\}) \ \& \ (\{\cap\}(\mathbf{X}) \triangleq \{ \bigcap_{X \in \mathcal{X}} X : \ \mathcal{X} \in \mathcal{P}'(\mathbf{X})\}) \ \& (\{\cup\}_\sharp(\mathbf{X}) \triangleq \{ \bigcup_{X \in \mathcal{K}} X : \ \mathcal{K} \in \mathrm{Fin}(\mathbf{X})\}).$$

If U and V are sets, then by V^U we denote the set of all mappings from U into V; if $g \in V^U$ and $W \in \mathcal{P}(U)$, then

$$g^1(W) \triangleq \{g(w) : \ w \in W\} \in \mathcal{P}(V)$$

is the image of W under operation of g. For any family $\mathcal{H}$ and a set S, we suppose that

$$[\mathcal{H}](S) \triangleq \{H \in \mathcal{H} | S \subset H\}.$$

Special families. In this subsection, we fix a nonempty set $\mathbf{I}$. Elements of the family $\mathcal{P}'(\mathcal{P}(\mathbf{I}))$ are nonempty families of subsets of $\mathbf{I}$;

$$\pi[\mathbf{I}] \triangleq \{\mathcal{I} \in \mathcal{P}'(\mathcal{P}(\mathbf{I})) | (\emptyset \in \mathcal{I}) \& (\mathbf{I} \in \mathcal{I}) \& (\ A \cap B \in \mathcal{I} \ \ \forall A \in \mathcal{I} \ \forall B \in \mathcal{I})\}. \tag{2}$$

Elements of (2) are π-systems in $\mathbf{I}$ with "zero" and "unit";

$$\tilde{\pi}^0[\mathbf{I}] \triangleq \{\mathcal{I} \in \pi[\mathbf{I}] | \ \forall L \in \mathcal{I} \ \forall x \in \mathbf{I} \setminus L \ \exists \Lambda \in \mathcal{I} : \ (x \in \Lambda) \& (\Lambda \cap L = \emptyset)\}$$

is the family of all separable π-systems of $\pi[\mathbf{I}]$. By $\pi^\natural[\mathbf{I}]$ we denote the family of all $\mathcal{I} \in \pi[\mathbf{I}]$ for which, under any set $L \in \mathcal{I}$, the intersection of all sets of $[\mathbf{C_I}[\mathcal{I}]](L)$ is an element of $\mathbf{C_I}[\mathcal{I}]$. We introduce the family of all lattices in the set $\mathbf{I}$:

$$(\mathrm{LAT})_0[\mathbf{I}] \triangleq \{\mathcal{L} \in \pi[\mathbf{I}] | \ A \cup B \in \mathcal{L} \ \forall A \in \mathcal{L} \ \forall B \in \mathcal{L}\} \tag{3}$$

(we consider only lattices with "zero" and "unit"). Of course,

$$(\mathrm{alg})[\mathbf{I}] \triangleq \{\mathcal{A} \in (\mathrm{LAT})_0[\mathbf{I}] | \mathbf{I} \setminus A \in \mathcal{A} \ \forall A \in \mathcal{A}\},$$

$$(\mathrm{top})[\mathbf{I}] \triangleq \{\tau \in (\mathrm{LAT})_0[\mathbf{I}] | \bigcup_{G \in \mathcal{G}} G \in \tau \ \forall \mathcal{G} \in \mathcal{P}'(\tau)\}, \tag{4}$$

$$(\mathrm{clos})[\mathbf{I}] \triangleq \{\mathcal{F} \in (\mathrm{LAT})_0[\mathbf{I}] | \bigcap_{F \in \mathcal{F}'} F \in \mathcal{F} \ \forall \mathcal{F}' \in \mathcal{P}'(\mathcal{F})\}; \tag{5}$$

by (4) topologies on $\mathbf{I}$ are defined. Under $\tau \in (\mathrm{top})[\mathbf{I}]$, by $(\mathbf{I}, \tau)$ a topological space (TS) is realized; if $S \in \mathcal{P}(E)$, then by $\mathrm{cl}(S, \tau)$ we denote the closure of S in the sense of TS $(\mathbf{I}, \tau)$. Elements of the families (4) and (5) are in a natural duality. We obtain the wide spectrum of concrete variants of π-systems. Under $\mathcal{I} \in \pi[\mathbf{I}]$, in the form of

$$(\mathrm{Cen})[\mathcal{I}] \triangleq \{\mathcal{Z} \in \mathcal{P}'(\mathcal{I}) | \bigcap_{Z \in \mathcal{K}} Z \neq \emptyset \ \forall \mathcal{K} \in \mathrm{Fin}(\mathcal{Z})\},$$

we obtain the family of all centered subfamilies of $\mathcal{I}$.

The family $\mathcal{H} \in \mathcal{P}'(\mathcal{P}(\mathbf{I}))$ is called linked in the case when $A \cap B \neq \emptyset \ \forall A \in \mathcal{H} \ \forall B \in \mathcal{H}$. Let

$$(\mathrm{link})[\mathbf{I}] \triangleq \{\mathcal{I} \in \mathcal{P}'(\mathcal{P}(\mathbf{I})) | \Sigma_1 \cap \Sigma_2 \neq \emptyset \ \forall \Sigma_1 \in \mathcal{I} \ \forall \Sigma_2 \in \mathcal{I}\}.$$

It is useful to consider linked subfamilies of a fixed family. So, under $\mathbf{X} \in \mathcal{P}'(\mathcal{P}(\mathbf{I}))$, we suppose that $\langle \mathbf{X} - \mathrm{link} \rangle[\mathbf{I}] \triangleq \{\mathcal{I} \in (\mathrm{link})[\mathbf{I}] | \mathcal{I} \subset \mathbf{X}\}$ and

$$\langle \mathbf{X} - \mathrm{link} \rangle_0[\mathbf{I}] \triangleq \{\mathcal{X} \in \langle \mathbf{X} - \mathrm{link} \rangle[\mathbf{I}] | \forall \mathcal{Y} \in \langle \mathbf{X} - \mathrm{link} \rangle[\mathbf{I}] \ (\mathcal{X} \subset \mathcal{Y}) \Longrightarrow (\mathcal{X} = \mathcal{Y})\}. \tag{6}$$

Elements of (6) are maximal linked systems (MLS) in class of subfamilies of $\mathbf{X}$. If $\chi \in \mathcal{P}'(\mathcal{P}(\mathbf{I}))$, then

$$(\mathrm{COV}[\mathbf{I}; \chi] \triangleq \{\kappa \in \mathcal{P}'(\chi) | \mathbf{I} = \bigcup_{X \in \kappa} X\}.$$

Now, we suppose that

$$(\mathrm{p-BAS})[\mathbf{I}] \triangleq \{\eta \in \mathcal{P}'(\mathcal{P}(\mathbf{I})) | \mathbf{I} = \bigcup_{J \in \eta} J\}$$

(the family of all open subbasises of $\mathbf{I}$ is introduced). Then, under $\eta \in (\mathrm{p-BAS})[\mathbf{I}]$, in the form of $\{\cap\}_\sharp(\eta)$, the base of some topology on $\mathbf{I}$ is realized; this topology is $\{\cup\}(\{\cap\}_\sharp(\eta))$. If $\tau \in (\mathrm{top})[\mathbf{I}]$, then

$$(\mathrm{p-BAS})_0[\mathbf{I}; \tau] \triangleq \{\eta \in (\mathrm{p-BAS})[\mathbf{I}] | \tau = \{\cup\}(\{\cap\}_\sharp(\eta))\}$$

is the family of all open subbasises of TS $(\mathbf{I}, \tau)$. Now, we introduce supercompact topologies:

$$((\mathbf{SC}) - \mathrm{top})[\mathbf{I}] \triangleq \{\tau \in (\mathrm{top})[\mathbf{I}] | \exists \mathcal{S} \in (\mathrm{p-BAS})_0[\mathbf{I}; \tau] \ \forall \mathcal{G} \in (\mathrm{COV})[\mathbf{I}; \mathcal{S}] \ \exists G_1 \in \mathcal{G} \ \exists G_2 \in \mathcal{G} : \ \mathbf{I} = G_1 \cup G_2\}$$

(see in (A.G.Chentsov , 2018, (5.2)) equivalent representation in terms of closed binary subbasises). If $\tau \in ((\mathbf{SC}) - \mathrm{top})[\mathbf{I}]$, then TS $(\mathbf{I}, \tau)$ is called supercompact; moreover, if $(\mathbf{I}, \tau)$ is a T_2-space, then $(\mathbf{I}, \tau)$ is called supercompactum. We recall that every supercompact TS is a compact TS.

Now, we fix $\mathcal{J} \in \pi[\mathbf{I}]$. We introduce

$$\mathbf{F}^*(\mathcal{J}) \triangleq \{\mathcal{F} \in \mathcal{P}'(\mathcal{J}) | (\emptyset \notin \mathcal{F}) \& (A \cap B \in \mathcal{F} \ \ \forall A \in \mathcal{F} \ \forall B \in \mathcal{F}) \& (\forall F \in \mathcal{F} \ \forall J \in \mathcal{J} \ \ (F \subset J) \Longrightarrow (J \in \mathcal{F})\}$$

(the family of all filters of space $(\mathbf{I}, \mathcal{J})$) and

$$\begin{aligned}\mathbf{F}_0^*(\mathcal{J}) \triangleq \{\mathcal{U} \in \mathbf{F}^*(\mathcal{J}) | \forall \mathcal{V} \in \mathbf{F}^*(\mathcal{J})\ (\mathcal{U} \subset \mathcal{V}) \Longrightarrow (\mathcal{U} = \mathcal{V})\} \\ = \{\mathcal{U} \in \langle \mathcal{J} - \mathrm{link}\rangle_0[\mathbf{I}] | A \cap B \in \mathcal{U} \forall A \in \mathcal{U}\ \forall B \in \mathcal{U}\} \\ = \{\mathcal{U} \in (\mathrm{Cen})[\mathcal{J}] | \forall \mathcal{V} \in (\mathrm{Cen})[\mathcal{J}] \\ (\mathcal{U} \subset \mathcal{V}) \Longrightarrow (\mathcal{U} = \mathcal{V})\};\end{aligned} \tag{7}$$

$\mathbf{F}_0^*(\mathcal{J}) \neq \emptyset$. Elements of (7) are ultrafilters of space $(\mathbf{I}, \mathcal{J})$. In addition, under $x \in \mathbf{I}$, we obtain that $(\mathcal{J} - \mathrm{triv})[x] \triangleq \{J \in \mathcal{J} | x \in J\} \in \mathbf{F}^*(\mathcal{J})$. Then, by (A.G.Chentsov , 2014, (5.9))

$$((\mathcal{J} - \mathrm{triv})[x] \in \mathbf{F}_0^*(\mathcal{J})\ \forall x \in \mathbf{I}) \Longleftrightarrow (\mathcal{J} \in \tilde{\pi}^0[\mathbf{I}]).$$

3. TOPOLOGIES OF SPACES OF ULTRAFILTERS AND MAXIMAL LINKED SYSTEMS

Fix a nonempty set E and $\mathcal{L} \in \pi[E]$. Suppose that $\Phi_{\mathcal{L}}(L) \triangleq \{\mathcal{U} \in \mathbf{F}_0^*(\mathcal{L}) | L \in \mathcal{U}\}\ \forall L \in \mathcal{L}$. Then

$$(\mathbf{UF})[E; \mathcal{L}] \triangleq \{\Phi_{\mathcal{L}}(L) :\ L \in \mathcal{L}\} \in \pi[\mathbf{F}_0^*(\mathcal{L})]$$

and, in particular, $(\mathbf{UF})[E; \mathcal{L}]$ is a base of topology $\mathbf{T}_{\mathcal{L}}^*[E] \triangleq \{\cup\}((\mathbf{UF})[E; \mathcal{L}]) \in (\mathrm{top})[\mathbf{F}_0^*(\mathcal{L})]$. In addition,

$$(\mathbf{F}_0^*(\mathcal{L}), \mathbf{T}_{\mathcal{L}}^*[E]) \tag{8}$$

is a zero-dimensional T_2-space for which

$$(\mathbf{UF})[E; \mathcal{L}] \subset \mathbf{T}_{\mathcal{L}}^*[E] \cap \mathbf{C}_{\mathbf{F}_0^*(\mathcal{L})}[\mathbf{T}_{\mathcal{L}}^*[E]] \tag{9}$$

(if $\mathcal{L} \in (\mathrm{alg})[E]$, then (8) is Stone compactum and (9) is converted in equality).

Now, consider another scheme of topological equipment for $\mathbf{F}_0^*(\mathcal{L})$. Namely, we suppose that

$$\mathbf{F}_{\mathbf{C}}[\mathcal{L}|H] \triangleq \{\mathcal{U} \in \mathbf{F}_0^*(\mathcal{L}) | \exists U \in \mathcal{U} :\ U \subset H\}\ \forall H \in \mathcal{P}(E). \tag{10}$$

Of course, $\mathbf{F}_{\mathbf{C}}[\mathcal{L}|E \setminus L] = \mathbf{F}_0^*(\mathcal{L}) \setminus \Phi_{\mathcal{L}}(L)$ under $L \in \mathcal{L}$. Then

$$\mathcal{F}_{\mathbf{C}}[\mathcal{L}] \triangleq \{\mathbf{F}_{\mathbf{C}}[\mathcal{L}|\Lambda] :\ \Lambda \in \mathbf{C}_E[\mathcal{L}]\} = \mathbf{C}_{\mathbf{F}_0^*(\mathcal{L})}[(\mathbf{UF})[E; \mathcal{L}]]$$

is a base of closed sets in TS (8):

$$\mathbf{C}_{\mathbf{F}_0^*(\mathcal{L})}[\mathbf{T}_{\mathcal{L}}^*[E]] = \{\cap\}(\mathcal{F}_{\mathbf{C}}[\mathcal{L}]).$$

By (A.G.Chentsov , 2018, Proposition 3.1) $\forall L \in \mathcal{L}\ \forall \Lambda \in \mathbf{C}_E[\mathcal{L}]$

$$(L \subset \Lambda) \Longleftrightarrow (\Phi_{\mathcal{L}}(L) \subset \mathbf{F}_{\mathbf{C}}[\mathcal{L}|\Lambda]).$$

As a corollary, under $L \in \mathcal{L}$, we have $[\mathcal{F}_{\mathbf{C}}[\mathcal{L}]](\Phi_{\mathcal{L}}(L)) = \{\mathbf{F}_{\mathbf{C}}[\mathcal{L}|\Lambda] :\ \Lambda \in [\mathbf{C}_E[\mathcal{L}]](L)\}$ and

$$\Phi_{\mathcal{L}}(L) = \bigcap_{\Lambda \in [\mathbf{C}_E[\mathcal{L}]](L)} \mathbf{F}_{\mathbf{C}}[\mathcal{L}|\Lambda].$$

With employment of this representation, the next statement is established.

Proposition 1. The following equality is valid:

$$\begin{aligned}\mathbf{F}_0^*(\mathcal{L}) = \{\mathcal{U} \in \mathbf{F}^*(\mathcal{L}) | \forall L \in \mathcal{L}\ \ (L \in \mathcal{U}) \\ \vee (\exists \Lambda \in [\mathbf{C}_E[\mathcal{L}]](L) :\ E \setminus \Lambda \in \mathcal{U})\}.\end{aligned}$$

We note that, under $\mathcal{L} \in \pi^\natural[E]$

$$(\mathbf{UF})[E; \mathcal{L}] \subset \mathcal{F}_{\mathbf{C}}[\mathcal{L}]. \tag{11}$$

Moreover, we note that (see (A.G.Chentsov , 2014, (5.4))) $\mathcal{F}_{\mathbf{C}}[\mathcal{L}]$ is (open) subbasis of topology

$$\mathbf{T}_{\mathcal{L}}^0\langle E\rangle \triangleq \{\cup\}(\{\cap\}_\sharp(\mathcal{F}_{\mathbf{C}}[\mathcal{L}])) \in (\mathrm{top})[\mathbf{F}_0^*(\mathcal{L})]. \tag{12}$$

In addition (A.G.Chentsov , 2014, Section 7), the following inclusion

$$\mathbf{T}_{\mathcal{L}}^0\langle E\rangle \subset \mathbf{T}_{\mathcal{L}}^*[E] \tag{13}$$

is realized; moreover, with employment of (11), we obtain that

$$(\mathcal{L} \in \pi^\natural[E]) \Longrightarrow (\mathbf{T}_{\mathcal{L}}^0\langle E\rangle = \mathbf{T}_{\mathcal{L}}^*[E]). \tag{14}$$

In addition, $(\mathrm{alg})[E] \subset \pi^\natural[E]$ and $(\mathrm{top})[E] \subset \pi^\natural[E]$. So, for these particular cases, we obtain (see A.G.Chentsov (2018)) the coincidence of topologies in (13). Of course, for $\mathcal{L} \in \pi^\natural[E]$, topology $\mathbf{T}_{\mathcal{L}}^0\langle E\rangle = \mathbf{T}_{\mathcal{L}}^*[E]$ converts $\mathbf{F}_0^*(\mathcal{L})$ in zero-dimensional compactum.

Remark 1. Now, we consider the case $\mathcal{L} = \mathbf{C}_E[\tau]$, where $\tau \in (\mathrm{top})[E]$ and (E, τ) is a T_1-space. Then (see A.G.Chentsov (2018); A.G.Chentsov (2017)), under $\mathcal{L} \neq \mathcal{P}(E)$, the property $\mathbf{T}_{\mathcal{L}}^0\langle E\rangle \neq \mathbf{T}_{\mathcal{L}}^*[E]$ is realized. In A.G.Chentsov (2017), some informative cases of such situation are reduced. □

We note that, in general case $\mathcal{L} \in \pi[E]$

$$(\mathbf{F}_0^*(\mathcal{L}), \mathbf{T}_{\mathcal{L}}^0\langle E\rangle) \tag{15}$$

is a compact T_1-space. We note that for $\mathcal{L} \in \tilde{\pi}^0[E]$

$$\begin{aligned}\Phi_{\mathcal{L}}(L) = \mathrm{cl}(\{(\mathcal{L} - \mathrm{triv})[x] :\ x \in L\}, \mathbf{T}_{\mathcal{L}}^0\langle E\rangle) \\ = \mathrm{cl}(\{(\mathcal{L} - \mathrm{triv})[x] :\ x \in L\}, \mathbf{T}_{\mathcal{L}}^*[E])\ \forall L \in \mathcal{L}.\end{aligned}$$

Now, in general case $\mathcal{L} \in \pi[E]$, we note some properties of MLS. In this constructions, we follow to A.G.Chentsov (2018). Introduce the sets

$$\begin{aligned}(\langle \mathcal{L} - \mathrm{link}\rangle^0[E|L] \triangleq \{\mathcal{E} \in \langle \mathcal{L} - \mathrm{link}\rangle_0[E] | \\ L \in \mathcal{E}\}\ \forall L \in \mathcal{L})\&(\langle \mathcal{L} - \mathrm{link}\rangle_{\mathrm{op}}^0[E|H] \\ \triangleq \{\mathcal{E} \in \langle \mathcal{L} - \mathrm{link}\rangle_0[E] | \exists \Sigma \in \mathcal{E} :\ \Sigma \subset H\} \\ \forall H \in \mathcal{P}(E)).\end{aligned}$$

In terms of these sets, we introduce two following families:

$$\begin{aligned}(\hat{\mathcal{C}}_0^*[E; \mathcal{L}] \triangleq \{\langle \mathcal{L} - \mathrm{link}\rangle^0[E|L] :\ L \in \mathcal{L}\}) \\ \&(\hat{\mathcal{C}}_{\mathrm{op}}^0[E; \mathcal{L}] \triangleq \{\langle \mathcal{L} - \mathrm{link}\rangle_{\mathrm{op}}^0[E|\Lambda] :\ \Lambda \in \mathbf{C}_E[\mathcal{L}]\}).\end{aligned}$$

The last family is a subbasis of a topology:

$$\hat{\mathcal{C}}_{\mathrm{op}}^0[E; \mathcal{L}] \in (\mathrm{p} - \mathrm{BAS})[\langle \mathcal{L} - \mathrm{link}\rangle_0[E]].$$

This topology is defined by the rule

$$\mathbf{T}_0\langle E|\mathcal{L}\rangle \triangleq \{\cup\}(\{\cap\}_\sharp(\hat{\mathcal{C}}_{\mathrm{op}}^0[E; \mathcal{L}])) \in (\mathrm{top})[\langle \mathcal{L} - \mathrm{link}\rangle_0[E]].$$

And what is more, we obtain that $\mathbf{T}_0\langle E|\mathcal{L}\rangle \in ((\mathbf{SC}) - \mathrm{top})[\langle \mathcal{L} - \mathrm{link}\rangle_0[E]]$, and

$$(\langle \mathcal{L} - \mathrm{link}\rangle_0[E], \mathbf{T}_0\langle E|\mathcal{L}\rangle) \tag{16}$$

is a supercompact T_1-space for which the property $\mathbf{T}_{\mathcal{L}}^0\langle E\rangle = \mathbf{T}_0\langle E|\mathcal{L}\rangle|_{\mathbf{F}_0^*(\mathcal{L})}$ takes place. So, $(\mathbf{F}_0^*(\mathcal{L}), \mathbf{T}_{\mathcal{L}}^0\langle E\rangle)$ (15) is a subspace of TS (16). We obtain two TS defined by Wallman scheme. Now, consider the Stone variant of topological equipment. We use the obvious property

$$\hat{\mathcal{C}}_0^*[E; \mathcal{L}] \in (\mathrm{p} - \mathrm{BAS})[\langle \mathcal{L} - \mathrm{link}\rangle_0[E]].$$

As a corollary, we obtain topology

$$\mathbf{T}_*\langle E|\mathcal{L}\rangle \triangleq \{\cup\}(\{\cap\}_\sharp(\hat{\mathcal{C}}_0^*[E; \mathcal{L}])) \in (\mathrm{top})[\langle \mathcal{L} - \mathrm{link}\rangle_0[E]]$$

for which

$$(\langle \mathcal{L} - \mathrm{link}\rangle_0[E], \mathbf{T}_*\langle E|\mathcal{L}\rangle) \tag{17}$$

is a zero-dimensional T_2-space. In addition, $\mathbf{T}_{\mathcal{L}}^*[E] = \mathbf{T}_*\langle E|\mathcal{L}\rangle|_{\mathbf{F}_0^*(\mathcal{L})}$. So, (8) is a subspace of TS (17). We recall that $\mathbf{T}_0\langle E|\mathcal{L}\rangle \subset \mathbf{T}_*\langle E|\mathcal{L}\rangle$. This property is coordinated with (13). Moreover, if $\mathcal{L} \in (\text{alg})[E]$ or $\mathcal{L} \in (\text{top})[E]$, then we obtain unit topology $\mathbf{T}_0\langle E|\mathcal{L}\rangle = \mathbf{T}_*\langle E|\mathcal{L}\rangle$ which converts $\langle \mathcal{L} - \text{link}\rangle_0[E]$ in zero-dimensional supercompactum. In general case $\mathcal{L} \in \pi[E]$, in the form of

$$(\langle \mathcal{L} - \text{link}\rangle_0[E], \mathbf{T}_0\langle E|\mathcal{L}\rangle, \mathbf{T}_*\langle E|\mathcal{L}\rangle),$$

bitopological space of MLS is realized.

4. ATTRACTION SETS

Consider natural problem connected with attainability in TS under constraints of asymptotic character. So, we fix a T_2-space (a Hausdorff TS) $(\mathbf{H}, \tau)$, $\mathbf{H} \neq \emptyset$, and a mapping $\mathbf{h} \in \mathbf{H}^E$. We consider $\mathbf{h}$ as goal operator. In E, we introduce a (nonempty) family $\mathcal{E} \in \mathcal{P}'(\mathcal{P}(E))$ with the property $\forall \Sigma_1 \in \mathcal{E}\ \forall \Sigma_2 \in \mathcal{E}\ \exists \Sigma_3 \in \mathcal{E}:$

$$\Sigma_3 \subset \Sigma_1 \cap \Sigma_2.$$

So, $\mathcal{E}$ is a directed family. Then

$$(\text{AS})[\mathcal{E}] \triangleq \bigcap_{\Sigma \in \mathcal{E}} \text{cl}(\mathbf{h}^1(\Sigma), \tau) \in \mathbf{C}_{\mathbf{H}}[\tau] \tag{18}$$

is attraction set corresponding to our directed family $\mathcal{E}$. This family defines constraints of asymptotic character. We consider (18) as basic attraction set. For construction of (18), we use scheme of A.G.Chentsov (2016) connected with employment of auxiliary attraction set. So, if (K, θ), $K \neq \emptyset$, is a compact TS, $m \in K^E$, and $g \in \mathbf{H}^K$ is continuous mapping in the sense of (θ, τ) with the property $\mathbf{h} = g \circ m$, then we call (K, θ, m, g) a compactifier. For any compactifier $(\mathbf{K}, \tilde{\mathbf{t}}, p, q)$ by analogy with (18) the (auxiliary) attraction set is defined: we consider the set

$$(\text{as})[\mathcal{E}] \triangleq \bigcap_{\Sigma \in \mathcal{E}} \text{cl}(p^1(\Sigma), \tilde{\mathbf{t}}) \in \mathbf{C}_{\mathbf{K}}[\tilde{\mathbf{t}}]; \tag{19}$$

in addition, the following equality is realized:

$$(\text{AS})[\mathcal{E}] = q^1((\text{as})[\mathcal{E}]). \tag{20}$$

Many options of compactifiers are known. We confine ourselves to a discussion connected with ultrafilters using. In this connection, we remind constructions of (A.G.Chentsov , 2018, Section 7). So, later we will suppose that E is equipped with the topology $\mathbf{t} \in (\text{top})[E]$ for which $(E, \mathbf{t})$ is a T_1-space. We introduce $\mathbf{C}_E[\mathbf{t}] \in (\text{LAT})_0[E]$ and suppose (later) that $\mathcal{L} = \mathbf{C}_E[\mathbf{t}]$ unless otherwise contrary. But, now, we consider the more general case supposing only that $\mathcal{L} \in \tilde{\pi}^0[E]$. Of course, we obtain a variant of bitopological space

$$(\mathbf{F}_0^*(\mathcal{L}), \mathbf{T}_{\mathcal{L}}^0\langle E\rangle, \mathbf{T}_{\mathcal{L}}^*[E]). \tag{21}$$

We consider $(E, \mathcal{L})$ as a variant of widely understood measurable space. Of course, in our case, the mapping $(\mathcal{L} - \text{triv})[\cdot]$ defined as

$$x \longmapsto (\mathcal{L} - \text{triv})[x] : \ E \longrightarrow \mathbf{F}_0^*(\mathcal{L})$$

realizes immersion of E in the space (21). Suppose that $\mathcal{E} \subset \mathcal{L}$ and consider the set

$$\mathbf{F}_0^*(\mathcal{L} \mid \mathcal{E}) \triangleq \{\mathcal{U} \in \mathbf{F}_0^*(\mathcal{L}) | \mathcal{E} \subset \mathcal{U}\} \in \mathbf{C}_{\mathbf{F}_0^*(\mathcal{L})}[\mathbf{T}_{\mathcal{L}}^*[E]].$$

But, for our goals, another topological equipment is required. Namely, we use topology (12) for which compact T_1-space (15) is realized.

Proposition 2. In general case $\mathcal{L} \in \tilde{\pi}^0[E]$, for any $(\mathbf{T}_{\mathcal{L}}^0\langle E\rangle, \tau)$-continuous mapping $\mathbf{g} \in \mathbf{H}^{\mathbf{F}_0^*(\mathcal{L})}$ with the property $\mathbf{h} = \mathbf{g} \circ (\mathcal{L} - \text{triv})[\cdot]$,

$$(\mathbf{F}_0^*(\mathcal{L}), \mathbf{T}_{\mathcal{L}}^0\langle E\rangle, (\mathcal{L} - \text{triv})[\cdot], \mathbf{g}) \tag{22}$$

is a compactifiar.

The proof is obvious. Consider natural general variant of (19).

Proposition 3. Let $\mathcal{L} \in \tilde{\pi}^0[E]$ and the triplet $(\mathbf{K}, \tilde{\mathbf{t}}, p)$ of compactifiar used in (19) be defined as in Proposition 2: $\mathbf{K} = \mathbf{F}_0^*(\mathcal{L}),\ \tilde{\mathbf{t}} = \mathbf{T}_{\mathcal{L}}^0\langle E\rangle, p = (\mathcal{L} - \text{triv})[\cdot]$. Then,

$$(\text{as})[\mathcal{E}] = \mathbf{F}_0^*(\mathcal{L} \mid \mathcal{E}).$$

Proof. We note that $\mathbf{F}_0^*(\mathcal{L} \mid \mathcal{E})$ is intersection of all sets $\Phi_{\mathcal{L}}(\Sigma),\ \Sigma \in \mathcal{E}$. In addition, for every $\Sigma \in \mathcal{E}$ the equality $\Phi_{\mathcal{L}}(\Sigma) = \text{cl}(p^1(\Sigma), \mathbf{t})$ takes place (this property is used with employment of (A.G.Chentsov , 2011a, Proposition 7.1)). As a result, we obtain (see (19)) the required equality.

Remark 3.1. We note that (for $\mathcal{L} \in \tilde{\pi}^0[E]$)

$$\mathbf{F}_0^*(\mathcal{L} \mid \mathcal{E}) = \bigcap_{\Sigma \in \mathcal{E}} \text{cl}(\{(\mathcal{L} - \text{triv})[x] : \ x \in \Sigma\}, \mathbf{T}_{\mathcal{L}}^*[E]).$$

We obtain the simple corollary of Proposition 3: in general case of $\mathcal{L} \in \tilde{\pi}^0[E]$, for compactifier (22),

$$(\text{AS})[\mathcal{E}] = \mathbf{g}^1(\mathbf{F}_0^*(\mathcal{L} \mid \mathcal{E})).$$

Now, we consider a variant of above-mentioned general propositions using constructions similar Wallman extension. In addition, following to approach of (A.G.Chentsov and E.G. Pytkeev , 2014, Section 7), we suppose that $(\mathbf{H}, \tau)$ is a compactum (compact T_2-space).

Remark 3.2. Our additional requirement about compactness is natural for the case when some compactifier exists and scheme of type (20) can be used. Indeed, let (K, θ, m, g) be a compactifier. Then, (K, θ), $K \neq \emptyset$, is a compact TS, $m \in K^E$, $g \in \mathbf{H}^K$ is a (θ, τ)-continuous mapping, and $\mathbf{h} = g \circ m$. Then

$$\mathbf{h}^1(E) = g^1(m^1(E)) \subset g^1(K), \tag{23}$$

where $g^1(K)$ is a compact set in T_2-space $(\mathbf{H}, \tau)$. Of course, $g^1(K)$ is closed set in $(\mathbf{H}, \tau)$. Therefore, from (23), we obtain the inclusion

$$\tilde{\mathbf{H}} \triangleq \text{cl}(\mathbf{h}^1(E), \tau) \subset g^1(K).$$

Then, $\tilde{\mathbf{H}}$ is a nonempty compact set in $(\mathbf{H}, \tau) : \tilde{\tau} \triangleq \tau|_{\tilde{\mathbf{H}}}$ converts $\tilde{\mathbf{H}}$ in a compact TS. Moreover, $(\tilde{\mathbf{H}}, \tilde{\tau})$ is a T_2-space (indeed, $(\tilde{\mathbf{H}}, \tilde{\tau})$ is a subspace of T_2-space $(\mathbf{H}, \tau)$). So, $(\tilde{\mathbf{H}}, \tilde{\tau})$ is a nonempty compactum. We note that $\mathbf{h}^1(E) \subset \tilde{\mathbf{H}}$; therefore, $\mathbf{h} \in \tilde{\mathbf{H}}^E$ and $\mathbf{h}^1(\Sigma) \subset \tilde{\mathbf{H}}$ under $\Sigma \in \mathcal{E}$; as a corollary,

$$\text{cl}(\mathbf{h}^1(\Sigma), \tilde{\tau}) = \text{cl}(\mathbf{h}^1(\Sigma), \tau) \cap \tilde{\mathbf{H}} = \text{cl}(\mathbf{h}^1(\Sigma), \tau)$$

since $\Sigma \subset E$. Therefore, from (18), we obtain that

$$(\text{AS})[\mathcal{E}] = \bigcap_{\Sigma \in \mathcal{E}} \text{cl}(\mathbf{h}^1(\Sigma), \tilde{\tau}). \tag{24}$$

So, in considered case of compactified problem (we keep in mind the case when a compactifier exists), it is possible to replace the initial space $(\mathbf{H}, \tau)$ by compactum $(\tilde{\mathbf{H}}, \tilde{\tau})$ preserving attraction set (see (24)). □

So, our assumption about compactness of $(\mathbf{H}, \tau)$ is realistic for problems allowing the use of compactifiers. Now, in this case, we introduce the corresponding variant of compactifier (22). For this, it is sufficient to indicate the required variant $\mathbf{g}$. We recall that $\mathcal{L} \in \tilde{\pi}^0[E]$ (now, we restrict oneself to this general condition). Therefore, the set $\mathbf{F}_{\lim}[E; \mathcal{L}; \mathbf{H}; \tau] \in \mathcal{P}'(\mathbf{H}^E)$ defined in (A.G.Chentsov , 2013, (5.3)) is realized.

Now, we return to the case when $\mathcal{L} = \mathbf{C}_E[\mathbf{t}]$, where $\mathbf{t} \in (\text{top})[E]$ and $(E, \mathbf{t})$ is a T_1-space. Suppose that $\mathbf{h}$ is a continuous mapping in the sense of topologies $\mathbf{t}$ and τ ($(\mathbf{t}, \tau)$-continuous mapping): $\mathbf{h}^{-1}(G) \in \mathbf{t}\ \forall G \in \tau$. Then, by (A.G.Chentsov and E.G. Pytkeev , 2014, (7.9)) $\mathbf{h} \in \mathbf{F}_{\lim}[E; \mathcal{L}; \mathbf{H}; \tau]$ and the mapping

$$\varphi_{\lim}[\mathbf{h}|\mathcal{L}] : \ \mathbf{F}_0^*(\mathcal{L}) \longrightarrow \mathbf{H} \tag{25}$$

of (A.G.Chentsov and E.G. Pytkeev , 2014, Section 7) is defined; in addition, by (A.G.Chentsov and E.G. Pytkeev , 2014, (7.14))

$$\varphi_{\lim}[\mathbf{h}|\mathcal{L}] \circ (\mathcal{L} - \text{triv})[\cdot] = \mathbf{h}. \tag{26}$$

In addition, by (A.G.Chentsov and E.G. Pytkeev , 2014, Proposition 7.1) we obtain that $\varphi_{\lim}[\mathbf{h}|\mathcal{L}]$ (25)) is the mapping continuous with respect to topologies $\mathbf{T}^0_{\mathcal{L}}\langle E\rangle$ and τ (we use (A.G.Chentsov , 2018, (8.4)). So, by (26) and compactness of TS (15) we obtain that

$$(\mathbf{F}_0^*(\mathcal{L}), \mathbf{T}^0_{\mathcal{L}}\langle E\rangle, (\mathcal{L} - \text{triv})[\cdot], \varphi_{\lim}[\mathbf{h}|\mathcal{L}])$$

is compactifier for which the corresponding variant of attraction set $(\text{as})[\mathcal{E}]$ (19) is the set (A.G.Chentsov and E.G. Pytkeev , 2014, (7.3)). So, for our case, we realize the mapping $\mathbf{g}$ in Propositions 2 and 3.As a result, (20) is reduced to (A.G.Chentsov and E.G. Pytkeev , 2014, Theorem 1):

$$(\text{AS})[\mathcal{E}] = \varphi_{\lim}[\mathbf{h}|\mathcal{L}]^1(\mathbf{F}_0^*(\mathcal{L}|\mathcal{E})). \tag{27}$$

So, the required attraction set (18) can be written in the form of concrete (A.G.Chentsov and E.G. Pytkeev , 2014, (7.14)) continuous image of the set $\mathbf{F}_0^*(\mathcal{L}|\mathcal{E})$ of admissible generalized elements.

5. ONE VARIANT OF ULTRAFILTER SPACE

Now, consider concrete variant of measurable space with algebra of sets for which the explicit description of ultrafilter space is known (see A.G.Chentsov (2011b)). This concrete representation was used in A.G.Chentsov and A.P.Baklanov (2015) for construction of attraction set in finite-dimensional space. With employment of this representation, in A.G.Chentsov, A.P. Baklanov, and I.I.Savenkov (2016), in control problem for mass point, the above-mentioned construction was realized in the form of program for PC. In addition, in A.G.Chentsov (2011b), constructions in the class of ultrafilters were used for obtaining the required description for generalized elements realized as finitely additive measures.

So, suppose that $E = [a, b]$, where $a \in \mathbf{R}$, $b \in \mathbf{R}$, and $a < b$. Let

$$\mathcal{J} \triangleq \{ L \in \mathcal{P}(E) | \exists c \in E\ \exists d \in E : \ (]c, d[\subset L) \& (L \subset [c, d]) \}$$

(the family of all intervals (open, half-open, and closed) contained in E). Suppose (in this section) that $\mathcal{A} \in (\text{alg})[E]$ is algebra of subsets of E generated by the family $\mathcal{J}$. In particular, $\mathcal{A} \in \tilde{\pi}^0[E]$. So, $(E, \mathcal{A})$ is a measurable space with algebra of sets. Let

$$(\mathcal{U}_t^{(-)} \triangleq \{A \in \mathcal{A} | \exists c \in [a, t[: \ [c, t[\subset A\} \ \forall t \in]a, b]) \ \& \ (\mathcal{U}_t^{(+)} \triangleq \{A \in \mathcal{A} | \exists c \in]t, b] :]t, c] \subset A\} \ \forall t \in [a, b[)$$

Then, by (A.G.Chentsov , 2011b, Proposition 6.1)

$$\mathbf{F}_0^*(\mathcal{A}) = \{\mathcal{U}_t^{(-)} : \ t \in]a, b]\} \cup \{\mathcal{U}_t^{(+)} : \ t \in [a, b[\} \cup \{(\mathcal{A} - \text{triv})[x] : \ x \in [a, b]\}.$$

So, we have exhausting description of the ultrafilter set for measurable space $(E, \mathcal{A})$. Moreover, we note that informative discussion of this description is reduced in (A.G.Chentsov , 2011b, Section 7).

We note that $\{x\} \in \mathcal{A}\ \forall x \in E$. Then, with employment of example 4.18 in (V.V. Fedorchuk and V.V. Filippov , 2006, Chapter 7), the following property is established:

$$\langle \mathcal{A} - \text{link}\rangle_0[E] \setminus \mathbf{F}_0^*(\mathcal{A}) \in \mathbf{T}_*\langle E|\mathcal{A}\rangle \setminus \{\emptyset\}. \tag{28}$$

As a corollary, $\mathbf{F}_0^*(\mathcal{A})$ is a closed set in TS $(\langle \mathcal{A} - \text{link}\rangle_0[E], \mathbf{T}_*\langle E|\mathcal{A}\rangle)$ for which $\mathbf{F}_0^*(\mathcal{A}) \neq \langle \mathcal{A} - \text{link}\rangle_0[E]$. We note the obvious property $\mathcal{A} \in \pi^\natural[E]$. Therefore, by (14) $\mathbf{T}^0_{\mathcal{A}}\langle E\rangle = \mathbf{T}^*_{\mathcal{A}}[E]$. Moreover, we obtain the equality $\mathbf{T}_0\langle E|\mathcal{A}\rangle = \mathbf{T}_*\langle E|\mathcal{A}\rangle$. As a corollary,

$$(\mathbf{F}_0^*(\mathcal{A}), \mathbf{T}^0_{\mathcal{A}}\langle E\rangle) = (\mathbf{F}_0^*(\mathcal{A}), \mathbf{T}^*_{\mathcal{A}}[E])$$

is a zero-dimensional compactum and

$$(\langle \mathcal{A} - \text{link}\rangle_0[E], \mathbf{T}_0\langle E|\mathcal{A}\rangle) = (\langle \mathcal{A} - \text{link}\rangle_0[E], \mathbf{T}_*\langle E|\mathcal{A}\rangle)$$

is zero-dimensional supercompactum. In addition, by (28)

$$\begin{aligned} &\text{cl}(\{(\mathcal{L} - \text{triv})[x] : \ x \in E\}, \mathbf{T}_0\langle E|\mathcal{A}\rangle) \\ &= \text{cl}(\{(\mathcal{L} - \text{triv})[x] : \ x \in E\}, \mathbf{T}^0_{\mathcal{A}}\langle E\rangle) = \mathbf{F}_0^*(\mathcal{A}). \end{aligned} \tag{29}$$

So, by (29) MLS of the (nonempty open) set (28) are not attainble in class of usual solutions (elements of E). In other words, in our case, all asymptotic effects are realized in class of ultrafilters.

6. CONCLUSION

In article, some constructions of topological character connected with possible realization of generalized elements for abstract control problems are considered. These constructions assume natural analogy with extension of TS although self objects in considered problems and in general topology are essentially different. In A.G.Chentsov and A.P.Baklanov (2015); A.G.Chentsov, A.P. Baklanov, and I.I.Savenkov (2016), for investigation of the control problem with impulse constraints, finitely additive measures are used as general controls. In representation of the corresponding admissible finitely additive measures, the description in terms of ultrafilters play the essential role (usefulness of ultrafilters for extension constructions is noted in other investigations also). It is naturally to investigate self ultrafilters (structure, topological properties and other). This investigation is realized in this article. Moreover, we consider the question about including space with respect to the ultrafilter space. This including space is induced: we consider maximal linked systems (MLS). In this connection, we note J. de Groot (1969); J. van. Mill (1977); M. Strok and A. Szymanski (1975). For this new space, natural topologies are introduced. For equipment of the set of MLS with the topology of Wallman type, supercompact space is realized.

We obtain two bitopological spaces. In addition, the bitopological space of ultrafilters (with Stone and Wallman topologies) is realized as a subspace of the bitopological space of MLS. The conditions of degeneracy and nondegeneracy are established. Later, constructions connected with attraction sets for abstract attainability problem are investigated (separately, Wallman compactifier is studied). Finally, one concrete variant of the ultrafilter space is considered (the case investigated in A.G.Chentsov (2011b)).

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